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Chapter 8

Dynamical Perspective on Motor Learning

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The construction of a theory of motor learning presupposes a theory of motor control. The latter specifies the syntax of the components that underlie the production of coordinated movements (e.g., reflexes and associations in reflex arcs, rules and representations in motor programs), whereas the former specifies how these components become reorganized (e.g., laws of association, hypothesis testing or induction, respectively) through interactions with the environment. This paper is a review of the dynamical theories of motor learning. To introduce this perspective, we will contrast the information-processing, connectionist, and dynamical theories of motor control in terms of their theoretical assumptions. Then we will review the dynamical theory of motor control or dynamical theory of action (as it is called) to provide a basis for understanding the dynamical theory of learning.

INFORMATION-PROCESSING, CONNECTIONIST, AND DYNAMICAL THEORIES OF BEHAVIORAL CONTROL

One can argue that any good theory of behavioral control should have two, not mutually exclusive, properties: (1) it should have theoretical and empirical continuity with its related sciences (biology, physics, chemistry); and (2) it should be non-circular—that is, it should not explain structure of a given complexity with components of equal complexity. Both connectionist (Bullock and Grossberg 1988; Churchland and Sejnowski 1989) and dynamical behavioral control (Turvey and Kugler 1984) theories have criticized contemporary information-processing theories of behavioral control along these lines.

The latter perspective (e.g., Schmidt, R.A. 1982a, 1982b) maintains that control is achieved because the actor has knowledge of a motor program—an internal representation that prescribes the activity of the effector components underlying the movement. The first criticism is based on how the assumptions of the existence of such an internal representation mesh with how the other sciences conceive the structure of action systems. The information-processing perspective on behavior is
based on positing a new domain of nature that is not continuous with the other sciences. Information processing rests on the existence of rule-governed, rate-independent processes that manipulate symbol strings (information), not unlike those found in systems of formal logic (Turvey and Kugler 1984). These rate-independent processes are to be contrasted with rate-dependent, law-based processes found in physical systems studied by other scientists. In fact, Newell and Simon's symbol string hypothesis (1971) suggests that symbolic, rate-independent formal processes are to be found nested within intelligent, physical systems. Further, it has been suggested that these logic processes are complementary to the dynamical, rate-dependent physical processes (Pattee 1972, 1977) investigated in the other sciences. Hence, the theoretical foundation of the information-processing view of behavior in general and the motor program view of motor control in particular is the philosophical premise of a new domain of nature in which the processes of intelligent behavior live. The criticism is that this drastic theoretical tactic destroys the continuity between the psychological and physical processes underlying behavior: A qualitatively different theory is required for understanding psychological processes.

The second criticism of the information-processing perspective and theory of motor programs rests on the isomorphic relationship between the knowledge structure posited (rules in a motor program) and the motor act it explains. Because rules are basic mechanisms for establishing a process and the postulating of a rule has very few constraints on it, what results in theorizing is taking loans of intelligence (Dennett 1971): Prescriptions for producing an action are proposed rather than principles that let the phenomena (e.g., actions) emerge "for free" as an a posteriori consequence of these principles. For example, explanatory concepts, such as motor programs and mental regression, needed for determining the parameters set in motor programs (Schmidt, R.A. 1975) are mechanisms that produce the motor behavior phenomena in question. But how these mechanisms relate to the other constraining details of the system and what deeper principles underlie their existence is not obvious and needs to be determined. The problem is whether such an explanation is forthcoming.

Churchland and Sejnowski (1989) think not and suggest that the information-processing theory of mental processes, the so-called Boolean dream (Hofstadter 1982), has the same form of explanation as the homuncular theory of reproduction, in which the structure of the developed neonate is posited as latent in the structure of the sperm. The problem with this kind of theory is that the key structure it is attempting to explain is posited in toto, without determining its origins or its ecology (i.e., how it fits into its context).

By contrast, connectionist theories explain behavioral control in a manner continuous with the biological structure of the computational machinery under study (i.e., the brain) and show how greater (behavioral) complexity emerges from the interaction of simpler biological structures. However, the power of connectionism comes not only from the lower-level complexity and biological nature of the processing units, but also from the processes written over them. In the connectionist theory, information processing is the result of a distributed interaction of microcomponents leading to the greatest constraint satisfaction (McClelland, Rumelhart, and Hinton 1986), or stated another way, states of the greatest harmony (Smolensky 1986). Churchland and Sejnowski (1989) point out that this processing mode is that of a dynamical system: "We model information processing in terms of the trajectory of a complex nonlinear system" (29). How is this? A dynamical system is a system whose components have an "interplay of forces" and "mutual influence" such that the system "tends towards equilibrium or steady states" (Kugler, Kelso, and Turvey 1980, 6). Hence, information processing in a connectionist system can be understood as dynamical and operating in concordance with physical organization strategies found generically in nature. The benefit of such a theory is that the dynamical information processing yields several properties required by any theory of cognitive systems (i.e., graceful degradation, generalization). Best of all, dynamical processing produces these properties for free, without needing specific mechanisms that exclusively produce these properties.

By the criteria for a good theory of behavioral control suggested previously, the connectionist theory is a good theory. It uses dynamical principles of organization found at all scales of nature to form its theory of control and demonstrates how more complex processes emerge from the interaction of simpler components. However, one can argue that the connectionist theory does not go far enough in investigating the dynamical principles underlying behavioral control (Schmidt, Treffner, and Turvey 1991). The argument is as follows. If dynamical principles of organization are generic archetypal themes that nature uses at all scales (Turvey 1990), we not only need to understand the dynamical processes of the nervous system, but also the dynamical processes that neural dynamics are nested within. The nervous system is nested within an action system and the action system within an environment. The perspective of the dynamical theory of behavioral control is that the dynamics of these higher order levels of structure constrain (and at some point determine) the structure of the neural dynamics. Hence, we need a theory of the dynamical processes at these levels and the neural level if we are going to have a complete theory of behavioral control.1

The organizations of behavioral systems that are more macroscopic than the level of neural dynamics are in the domain of the dynamical theory of action. It asks the following questions: What are the processes on the level of the action system and its interaction with the environment? What are the physical principles that are harnessed to organize the structured components at this scale of analysis? The dynamical theory of action answers: There are dynamical laws written in terms of the self-organizing, dynamical properties of the effector systems viewed as complex physical systems (Kugler, Kelso, and Turvey 1980; Kugler and Turvey 1987; Schöner and Kelso 1988); and there are laws written at a more macroscopic space/time scale that include the environmental properties and the information that specify them, namely, laws of perceiving and acting (Turvey and Carello 1988).

1Another criticism of the connectionist perspective is that many current models are general purpose. Many models can be used to generate any type of behavior. Assuming that all connectionist models implement a process of dynamical constraint satisfaction, the problem of being too general purpose is not an intrinsic consequence of the constraint-satisfaction process, but a consequence of choosing which constraints must be dynamically satisfied. One hopes that, with increasing influence from neuroscience and studies of perception and action, the constraints within connectionist models can become more ontologically specific and such models can become more special purpose.
to the laws of nature so they can be used to create the required organization. Further, to anticipate slightly, the dynamical perspective views learning new behaviors, not as acquiring rules of action encoded in the CNS, but acquiring new relationships to the laws of nature that underlie the action system and the way it relates to environmental properties (Shaw and Alley 1985).

**DYNAMICAL THEORY OF MOTOR CONTROL**

The organization of an action has been described as consisting of two stages. An organism first assembles an action and second guides the action to its completion. These two stages have been referred to as coordination and control, respectively (Kugler, Kelso, and Turvey 1980). Traditionally, two levels of explanation have been used to understand these processes—one psychological, one physiological. On the psychological or intentional level, the description is of the assembly or coordination of the action system with respect to a goal state or intentional directedness to some aspect of the environment. The control of an action is about applying rules of action based on the occurrence of environmental circumstances specified by perceptual information. On the physiological or machine level, the description is of the coordination of the action system with respect to efferent neural signals that appropriately potentiate the necessary effectors and their metabolic resources. In contrast the control of an action is defined in terms of an afferent tuning of the effector system states.

We can view the dynamical theory of action as positing a level of analysis embracing both these descriptions by a common language of explanation operating on the psychological and physiological levels—the language of dynamics. In place of the psychological and neural entities (e.g., motor programs, central pattern generators) traditionally proposed to understand coordination and control, the dynamical approach posits dynamical structures of control spread across several (neural, metabolic, biomechanical, informational, and environmental) levels of analysis and whose functioning is bound by dynamical principles of self-organization. Such a dynamical control regime is a functionally defined entity that describes the configuration of an organism's action system required to pursue a given goal. The evolution of this concept needs to be described.

Such control structures have been referred to over the years by several terms: coordinative structures (e.g., Easton 1972; Turvey 1977), functional synergies (e.g., Bernstein 1967), special-purpose devices (Fowler and Turvey 1978), or task-specific devices (e.g., Bingham 1988). Originally, Bernstein (1967) theorized that control

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Footnote: Maintaining that knowing how to perform an act does not require a mental prescription does not deny that some structural correlate exists tantamount to a representation in the knower, but takes issue with what this structural correlate is. There is no doubt that structural changes occur within the act where skill is learned. The question is what these changes consist of. The claim of the dynamical perspective is that the structural changes should not be viewed as the addition of a prescription of action components (i.e., a list of what to do when, encoded in symbol strings), but an emergent set of relations between CNS, action system, and environmental properties that form a dynamical system and manifest the action as an a posteriori consequence. Hence, what we propose is that what is learned should be understood as a set of physically encoded dynamical predispositions of an action system rather than a motor program.

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Footnote: Examples of these self-organizing tendencies at levels macroscopic to the CNS include the mass-springs properties of the action system and the omnipresent constraint of gravitational acceleration.
of an action system as complex as those in a biological organism was possible only because functional synergies—linkages of muscles and joints with each other and external reactive forces—were established in the effector performing a given action. Bernstein and his students maintained that the synergies consisted of equations of constraint written over the action system upon the assembly of the action. These equations of constraint specified dependencies of the controllable degrees of freedom on each other, reducing the total number of parameters to be controlled. Hence, the implementation of a synergy left the action system in many ways self-equilibrating and independent from central control.

**Dynamical Equations of Constraint**

Early research following the Bernstein approach discovered the functional synergies that underlie such coordinations as locomotion (Shik and Orlovskii 1976), aiming at a target (Arutyunyan, Gurfinikel, and Mirsky 1969), and maintaining an upright posture during breathing (Gurfinikel et al. 1971). A second round of Bernstein theorizing in the 1970s and 1980s (Turvey 1990) has sought a basis for equations of constraint in natural physical principles. The goal has been to find how much of the action system organization can be understood in terms of dynamical constraints, in terms of archetypal regimes of self-organization that occur in nature’s many scales (Turvey 1988). To understand this second round of Bernstein theorizing, we must explore what is meant by dynamical constraints or regimes.

A dynamical system is a general class of systems which have components that are mutually influencing and settle toward, or relax to equilibrium or attractive states. The study of qualitative dynamics (Abraham and Shaw 1982; Thompson and Stewart 1986) reveals that in nature, attractor states take a limited number of forms. In dynamical analysis of a given system, variables are chosen that represent the possible states the system can enter. The relations of these variables can be represented by geometric models of all possible states, called the state space of the system. A trajectory represents the evolution or history of a system’s states in a state space. Further, special trajectories are used to categorize the kind of dynamic the system displays. These trajectories that represent the asymptotic behavior of the system as time approaches infinity are called limit sets. The most probable limit sets are attractors. The simplest state space topology is a point attractor, in which all the trajectories within part of the state space are drawn to a single point. Another common topology is the periodic or limit cycle attractor in which the trajectories are attracted to a closed orbit. Higher order attractors with more complex limit set topologies also exist (e.g., quasiperiodic attractors, chaotic attractors). Though these higher order attractors can take many forms, they can be characterized by a few qualitative features that define them as integral classes of dynamical states.

A dynamical system can be represented by a state space plus a variable that represents a conserved quantity governing the time evolution of the system. This potential variable (or just potential, for short) underlies the flow of the system (i.e., the way that the trajectories evolve). Differences in the magnitude of the potential quantity over the state space will cause trajectories in the state space to flow to regions where the potential is less. The regions of the state space where the potential is least define regions of attraction and are the attractors of the system. The change rate of the potential surrounding the attractive area determines the stability or strength of an attractor. There are two means of measuring the stability or degree of attraction of an attractor. If one knows the potential function, $V(\alpha)$, which describes the potential field of an attractor over a single state variable $\alpha$, then $\frac{dV}{d\alpha}$ measures the degree of concavity of the attractor’s surrounding area (the potential well). The larger the $\frac{d^2V}{d\alpha^2}$, the more concave is the potential well, and the more stable or strong is the attractor (see fig. 8.2). Alternatively, stability can be measured empirically by the relaxation time of the system, $\tau_{eq}$—the time it takes the system to return to equilibrium following a perturbation. If one knows $\tau_{eq}$, the concavity of the potential well can be estimated ($\tau_{eq}$); further, if one knows the $\frac{d^2V}{d\alpha^2}$, one can estimate the relaxation time (Schöner 1989).

Using the tools of qualitative dynamics, researchers following Bernstein have provided several examples of coordinated movement in which dynamical principles of self-organization are operating. Three classes of coordinated movement have succumbed to a dynamical analysis. Discrete movements in which a limb segment is brought to a specific point in space have been characterized as having the dynamics of a point attractor (Feldman 1966a, b, 1986; Kugler, Kelso, and Turvey 1980; Schöner 1990). Rhythmic movements in which a limb segment is oscillated about a joint have been characterized as limit cycle oscillators having periodic attractors (Beek and Beek 1988; Kay et al. 1987; Kugler, Kelso, and Turvey 1980; Turvey et al. 1988). Further, coordinated interlimb rhythmic movements in which two limb segments are oscillated simultaneously about different joints have been characterized by coupled oscillator regimes that have point attractors at two relative phase angles corresponding to the in-phase ($0^\circ$) and anti-phase ($180^\circ$) modes of limb phasing (Haken, Kelso, and Bunz 1985; Schmidt et al. 1991; Schmidt, Shaw, and Turvey 1993; Sternad, Turvey, and Schmidt 1992). We will present examples of specific

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Footnote: Just what constitutes the state variable $\alpha$ is an empirical question. In the work on interlimb coordination, the state variables of the potential function models have been indices of the coordination (e.g., relative phasing or timing).
dynamical control structures that embody these regimes in a later section that explores the dynamical perspective's empirical research on motor learning.

Environmental Sources of Constraint

The second round of Bernstein theorizing has suggested that the equations of constraint underlying functional synergies make use of generic physical principles of self-organization, and that functional synergies can include in their organization noneffect physical properties. Classically, a functional synergy was a linkage of biomechanical properties constrained to produce a specific goal. Environmental influences on action coordination were deemed external and were viewed merely as inputs to action system control processes. Fowler and Turvey (1978) argued that environmental constraints specified by perceptual information must also be understood as part of the organization of action system synergies. They maintained that for each unconstrained degree of freedom in a muscle linkage, there was an environmental property to constrain its magnitude. The ramification of this proposal is that to understand action, one must understand the functioning of the effectors and the environmental context for the action—namely, those properties in the environment that a coordinated action must refer to in order to accomplish its final state.

Recent work on movement coordination with environmental events has led dynamical researchers to reevaluate where to find the boundary of the dynamical control structures. Research has found that coupled oscillator dynamics operate across a perceptual media (optic array) in the coordination of rhythmic movements between two people (Schmidt, Carello, and Turvey 1990; Schmidt and Turvey 1994) and between a person and an optical metronome (Wimmers, Beek, and van Wieringen 1992). Schmidt, Carello, and Turvey (1990) report that certain properties of dynamical change associated with the breakdown of the alternate phasing of limbs are found in between-person coordination (in which two persons coordinate the swinging of their lower legs) and in within-person bimanual coordination (e.g., Haken, Kelso, and Bunz 1985; Kelso, Scholz, and Schüner 1986). This work highlights the fact that anatomical or neural connections are not necessary to support dynamical control regimes—perceptual media can support these dynamical regimes as well.

The environmental constraints on the functioning of an action system, rather than being seen as an external influence on the dynamic, can be seen as a process intrinsic to the dynamics of the action system control structure. In a manner of speaking, an environmental constraint is one of the multiple constraints that must be dynamically satisfied. A dynamical control structure can contain both action system components and environmental properties specified by perceptual information. So, in terms of the self-organizing processes that underlie action organization, the boundary between the actor and the world functionally does not exist.

In sum, the dynamical theory of motor control maintains that underlying coordinated action are control structures that have the following characteristics. They are governed by the laws of self-organization, dynamical principles. The multiple constraints (effectors and environmental) on producing the action are mutually influencing, and their interaction brings dynamical steady states that satisfy these constraints. They are multileveled entities, in that multiple constraints occur at several nested levels—environmental, biomechanical, metabolic, and neural. These control structures are inherently informational entities because they are constrained by environmental properties specified by perceptual information. They are intentional, task-specific structures assembled to achieve a particular action goal. They are soft-assembled, temporary structures in that many constraints that coordinate the appropriate degrees of freedom are temporary and exist only until the goal or intention is fulfilled.

Finally, because of their dynamical nature, these control structures are relatively autonomous from central control in their functioning. One might say that the macroscopic processes of self-organization on the action system and environmental level leave the CNS processing open for other computations. Hence, the macroscopic, self-organizing nature of these control structures may be the basis for the lack of intentional intervention required to perform certain skillful activities. When performing skilled actions, the movements seem to be performed on their own. The perceptual tuning based on current environmental or action system circumstances appears automatic. By participating in the laws of nature, the burden of controlling one's movements is diminished. The apparent ease in performing a skillful action pattern, however, needs to be contrasted with the difficulty of learning a skillful action pattern. From the dynamical point of view, learning a skillful behavior is tantamount to becoming attuned to the laws of dynamical self-organization that govern an effector system. So, by participating in these laws the control of an action becomes effortless.

Dynamical Theory of Motor Learning

What is the process by which the laws of a dynamical control structure are learned so actions can be performed skillfully? If we consider assembling the appropriate functional synergy as the coordination of an action and the functioning of the synergy (maintaining equilibrium) its control, we can understand becoming skillful at producing the action as optimizing this synergy. Fowler and Turvey (1978) have suggested that "learning the laws means that the actor has become attuned (perceptually) to the consequences of different configurations of the multiple constraints on the control structure: An [action] has structure, and discovering an optimal self-organization is in reference to those variables of stimulation corresponding to environmental and biokinematic relations that specify the essential features of the [action] the actor is to perform" (6). They further suggest that such optimization corresponds to the increased use of the effectors' reactive forces, their interaction with the environment in action production (Bernstein 1967), and the increased number of degrees of freedom that are being controlled. The former process increases the control structure autonomy, whereas the latter increases the regime's stability and movement fluidity (Newell and van Emmerik 1989; van Emmerik and Newell 1990).

The description of the control structure optimization is similar to the optimization description in connectionist models of the CNS. We can understand the evolution of the control dynamic to more optimized states as increasing the number of "satisfied constraints" (cf. harmony, Smolensky 1986) that influence the control structure's functioning. The difference between the perspectives is that the constraint satisfaction
occurs on multiple levels of organization (neural, metabolic, biomechanic, and environmental) in the dynamical theory, rather than just the level of neural networks in connectionist theory.

By what process are these new constraints satisfied? What guides the actor to new configurations of his or her action system that are more optimal than old configurations? The dynamical perspective’s claim is that perceptual information is available that can guide the learner to more optimal states (Fowler and Turvey 1978; Kugler and Turvey 1987; Newell et al. 1989). The basis for this assertion is the ecological theory of perception (Gibson 1979). Any event (either in the environment or in an actor’s effector system) has both kinetic and kinematic consequences. An event’s kinetics (essentially the structure of its forces) generate kinematic patterns (a geometric representation of the force structure changing over time). If these patterns become manifest in perceptual (visual, auditory, haptic) media, they can be used as information that specifies the generating event and its properties. Much research (e.g., Bingham 1987; Runeson and Frykholm 1981) has demonstrated that kinematic invariants can specify properties of events and their future consequences (for example, time-to-contact (Lee 1980)). The kinematic patterns that specify the latter are particularly important for the prospective control of behavior—coordinating your movements with an event that is in the future.

The performance of an action by an action system is a kinetic event that generates kinematic patterns specific to the properties of the action in the visual and haptic/proprioceptive perceptual media. The kinematic patterns are information that specifies properties, such as the changing stability of the action system during the movement. Each implementation of a control structure (a certain equation of constraint) is characterized by kinematic/perceptual consequences that specify the stability of that execution. The initial implementations of a new action regime have dynamical equations of constraint that are nonstationary—they keep collapsing and need to be reassembled differently. Further implementations on subsequent attempts to produce the same action (e.g., in a learning experiment these are trials or sessions) will be more or less stable. With these stability changes, higher-order kinematic invariants are produced. This is information that specifies prospectively how the control structure needs to be reparameterized (i.e., how the equation of constraint needs to be changed) to increase its stability (i.e., constraint satisfaction) toward an optimally organized control regime (Fowler and Turvey 1978). Because of the reciprocity of action and perception in the evolution of an optimal action system organization, the learning process has been characterized as the exploration of a perceptual-motor work space of the nascent control regime (Kugler and Turvey 1987; Newell et al. 1989). In sum, the kinetics (forces) of a motor act generate consequent kinematic invariants in perceptual media. These kinematic patterns specify (a) how well the action is satisfying the task’s multiple constraints (its stability), and (b) how the control structure must be changed to increase the stability of the regime through increased constraint satisfaction.

Necessity of a Learning Dynamic

Note that to describe the acquisition of skilled behavior dynamically, one must describe two levels of change or two dynamics—one nested within the other. First we have the dynamic associated with the control structure assembled for producing the action—the coordinative structure or task-specific assembly of an effector system. This dynamic is set up when the action system is assembled under a particular dynamical equation of constraint. It exhibits a series of states that may or may not accomplish its intended goal, depending (more or less) on the optimality of the dynamical constraints employed. This action system control structure is nested within a higher-order dynamic that operates on it and represents the optimization of the control structure’s equation of constraint. We can view the functioning of a dynamical control structure and its optimization as two separate dynamical processes. This bipartite organization is analogous to activation rules and learning rules in connectionist models: The former specify how input signals are to be processed in each activation of the network, and the latter specify how the weights between processing units must be changed to process the input information more optimally.

How is this higher-order learning dynamic to be understood? First, it occurs at a longer time scale. If the control structure dynamic proceeds at a time scale of minutes and seconds, then the learning dynamic proceeds at a time scale of hours and days. The index of the former is the relaxation time \(\tau_{\text{rel}}\) of the control structure dynamic—the time it takes for the system to return to its equilibrium state from a perturbation. The index for the time scale of the latter is the relaxation time of the learning dynamic—the learning time \(\tau_{\text{learn}}\) (Schöner 1989). This latter quantity is the time it takes for optimization of the action system dynamic to occur. The phenomenon of learning to learn that occurs in learning several related skills would be tantamount to strengthening the learning dynamic, thereby decreasing the relaxation time of the learning dynamic \(\tau_{\text{learn}}\).

Second, the learning dynamic is intentional in that a future goal state determines the changes it undergoes. This future state is the state of being more skilled—to have an optimal equation of constraint governing the action system. Learning a skilled action must then be doubly intentional. Two intentions must be present: one specifies the goal of the action (e.g., catch the ball, stably produce a 3:2 polyrhythm) and another specifies the goal of the learning (i.e., become more skilled). Related to this point, these two intentions are temporally nested: The former occurs at a shorter time scale than the latter. One can then characterize learning a skill as two temporally nested intentional dynamics.

How does the future state of the system determine its present action or learning behavior? How does this acausal process function? In the traditional cognitive perspective, the future state is available now because it is an internal representation that contains the future in the present. How does the dynamical perspective account for this, given that it eschews internal representation? We have reviewed the answer in the discussion of the perceptual-motor work space.

The intention to become more skilled in producing action sets up a learning dynamic to optimize the lower-order action control structure (i.e., coordinative structure). The consequence of this learning dynamic is that information about this optimization process becomes available in the perceptual-motor work space, including information about the learning dynamic’s attractor—the optimal organization of the action system control structure. The kinematic flows created in the perceptual media (haptic, visual) can specify the attractor state of the system (e.g., the optimal state of an equation of constraint) even if that system is not at that state.
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\[ y(t) = kx(t) + \int_0^t k(t, s)x(s) \]  
(8.1)

where \( y(t) \) and \( x(t) \) represent the behavior and the state of the spring, respectively, at time \( t \), \( k \) is the spring's initial stiffness, \( k(t, s) \) is an operator that represents the hereditary influences on the stiffness and is a function of the pulls on the spring in the interval from 0 to \( t \). The behavior of the spring at any point in time \( t \) is a function of (a) the product of its original disposition \( k \) and present state \( x(t') \), plus (b) the integral of the hereditary changes in its behavior from the time of its original state to time \( t \).

The argument made by Shaw and colleagues is that models similar to this are necessary for modeling the optimization of an action control structure's functioning because they intrinsically capture the evolutionary nature of the system being modeled. As already noted, the structure of the Volterra equations captures the property that the evolving action systems have two nested dynamics—one at a fast time scale (state at time \( t \)) and one at a slow time scale (hereditary changes from 0 to \( t \)). That the hereditary influence represents a reinitialization of the system's equation of constraint (in this case, the mass-spring system's stiffness) allows one to draw the analogy between the hereditary influence and a learning dynamic that parameterizes the action control structure.

In addition, Shaw and colleagues suggest that these Volterra equations cannot only model learning based on historical feedback influences (i.e., increased constraint satisfaction resulting from experience performing the action), but also learning based on anticipatory feedforward influences (i.e., increased constraint satisfaction resulting from the anticipation of prospective future events). These "may manifest themselves as the cumulative effect of expectancies . . . that act on the current state of the learner" (Shaw et al. 1992, 7). In addition to providing a basis for modeling past and future influences on optimizing an action control structure, Shaw suggests that one must model the optimization of both the reactor input (afferent) processes of a dynamical control structure and the optimization of the energy control (efferent) processes. Accordingly, the strategy that he and his colleagues propose is no less than a system of Volterra equations that addresses the optimization of feedback, feedforward, afferent, and efferent subprocesses of a dynamical control structure (fig. 8.3).

Learning a New Bimanual Phasing Pattern

Schöner and colleagues (Schöner 1989; Schöner, Zanone, and Kelso 1992; Zanone and Kelso 1992) have modeled the learning dynamics involved in acquiring a specific skill—a novel bimanual phasing pattern. Bimanual phasing has two natural modes. Limbs can be easily coordinated so they are at the same place in the cycle at the same time (the in-phase mode or 0° relative phase) or so they are in opposite places in the cycle at the same time (the anti-phase mode or 180° relative phase). The two relative phase modes have been found differentially stable: The anti-phase mode is less stable than the in-phase. The bimodality and differential stability of bimanual phasing have been observed in experiments in which the 180° mode breaks down at higher frequencies of oscillation, whereas the 0° mode does not (Kelso 1984;
attainable by the coupled oscillatory regime. Fig. 8.4 demonstrates that these stable points of the regime appear at $\phi = 0^\circ$ and $\phi = 180^\circ$, and that the stable point at $\phi = 180^\circ$ is less stable (more shallow) than that at $0^\circ$. Increasing the oscillation frequency of the regime will lead to an annihilation of the $180^\circ$ attractor, assuming that the coupling strength $a$ decreases with increasing frequency, when the coupling strengths achieve the ratio of $b/a = 0.25$. In sum, this coupled oscillator dynamic predicts the bimodality and differential stability observed in bimanual limb phasing.

Such an equation of constraint represents the intrinsic dynamic of the bimanual rhythmic movement effector system and provides the foundational initial state that any new, to-be-learned bimanual phasing pattern would build on. Schöner’s modeling (1989) and Zanone and Kelso’s (1992) experiment capitalize on this in their investigations of learning a new relative phase pattern at $90^\circ$. They are interested in modeling and observing how the behavioral attractor layout evolves as a function of practicing a new relative phase relation at $90^\circ$.

In the empirical investigation, Zanone and Kelso employ a relative timing task (Tuller and Kelso 1989; Yamanishi, Kawato, and Suzuki 1980) in which the index fingers are paced by two visual metronomes, one for each finger. The frequency of the metronomes was fixed, but their relative timing was varied. The subject’s task was to flex each index finger in accord with the metronome specific to that hand. Thus, by manipulating relative timing of the two metronomes, the experimenter could control the bimanual relative phasing that the subject was to produce. Past research with the task has demonstrated that the intrinsic dynamic of bimanual phasing constrained subject performance (fig. 8.4). Subject performance was (a) best when the task requirement corresponded to one of the two intrinsic patterns (at $0^\circ$ or $180^\circ$ relative phase), (b) deviated in the direction of the intrinsic phase mode at other required relative phasing values, and (c) showed increasing variability as the required relative phase deviated from the intrinsic phase values.

Schmidt, Carello, and Turvey (1990) and in experiments that measure the degree of steady state fluctuation of the two modes (Turvey et al. 1986; Schmidt, Shaw, and Turvey 1993).

These properties have been explained as being a consequence of the dynamical nature of the control structure underlying bimanual coordination. The limb effector system acts identically to a regime of two coupled oscillators in which the coupling strength between the oscillators decreases with increasing frequency. Haken, Kelso, and Bunz (1985) have employed a coupled oscillator regime with point attractors at relative phase angles of $0^\circ$ and $180^\circ$ to model the bimanual phasing properties. The dynamical description of relative phase angle (\(\phi\)) between the two rhythmic units is defined as the differential equation:

\[
\dot{\phi} = -a \cos(\phi) - b \cos(2\phi) + \sqrt{Q\epsilon}
\]

(8.2)

where the rate of change of the relative phase angle (\(\dot{\phi}\)) is a function of the rate of change of the potential function, \(V(\Phi) = -a \cos(\phi) - b \cos(2\phi)\), and a stochastic noise term of magnitude \(Q\). The minima of the potential function (where its rate of change and, hence, \(\dot{\phi}\)'s rate of change is 0) indicate the stable relative phase patterns

\[V(\phi) = -a \cos(\phi) - b \cos(2\phi)\]

where \(a\) and \(b\) are a function of frequency

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Fig. 8.4 Haken, Kelso, and Bunz (1985) model of the attractors in bimanual, interlimb phasing. The potential function has attractor points (i.e., potential minima) at $0^\circ$ and $\pm180^\circ$ representing the in-phase and anti-phase interlimb phase modes, respectively. The minima at $\pm180^\circ$ are more shallow than at $0^\circ$, denoting the lesser stability of the anti-phase mode of interlimb phasing.

Adapted from Haken, Kelso, and Bunz 1985.
Zanon and Kelso were interested in whether having subjects practice a non-intrinsic relative phase pattern, in this case one at 90°, they could observe an evolution in the underlying attractor layout of the dynamical control structure at the practiced pattern from its initial intrinsic state to one with a new stable point. Note in fig. 8.4 that 90° relative phase is at an unstable fixed point—a dynamical repeller. A system initialized to this state will, as a consequence of small perturbations, relax to one of the two stable fixed points at 0° or 180°. Each subject practiced a 90° phasing pattern by tracking the visual metronome for three blocks of five learning trials on five consecutive days. Between learning sessions and at the beginning and end of each day, subjects performed scanning trials in which they had to track a changing relative phasing pattern (0° to 180° in 15° steps for 20 s) of the visual metronome in their bimanual movements. With this procedure, the stability of each relative phase pattern, as measured by the deviation from environmentally specified relative phase (φ - ψE) and the rate of change of φ (Δφ), could be evaluated throughout the learning period. The attractive (most stable) regions of the relative phase dynamic could be identified by this method.

Fig. 8.5 is a schematic that represents the changes in stability observed between the first and fifth days of the experiment. Initially, the intrinsic phase patterns (at 0° and 180°) were more stable and the to-be-learned pattern at 90° was least. However, by the fifth day, the practiced phase pattern at 90° was as stable as the intrinsic modes. In some subjects the increase in stability of the 90° pattern was yoked with a decrease in stability of the 180° pattern, suggesting that, not only was a new regime of stability attained, but an old regime of stability was lost.

How are these changes in the stability of relative phase patterns to be understood in terms of the dynamical model of relative phase control structure (Fig. 8.4)? Required is a model of the learning dynamic, the higher-order, long time scale dynamic that attracts the bimanual phasing toward the to-be-learned pattern specified by the environment (in this case, the visual metronome). Stated another way, we need a process that reconfigures the control structure governing the relative phasing to have an attractor state at this to-be-learned phase pattern. Schöner (1989) suggests that the learning dynamic in the present task can be represented by the following equation:

$$\dot{\psi} = -\tau_{learn}^{-1} \sin ((\phi - \psi_{0})/2)$$

(8.3)

where τ_{learn} is the time scale of the learning, ϕ is the present relative phase angle, and ψ_{0} is the to-be-learned relative phase angle. The change rate of acquiring the relative phase pattern (ψ) is a function of the difference between the present and the to-be-learned pattern and the intrinsic time scale of the learning. This latter magnitude, τ_{learn}, is set by experience with similar tasks (i.e., strength of learning to learn) or strength of information specifying ψ_{0}. The consequence of the interaction of the control structure dynamic (Equation 8.2) and the learning dynamic (Equation 8.3) can be represented by a potential function that combines the two dynamics (Zanone and Kelso 1992):

$$V_{\psi} = V(\phi) - c \cos((\phi - \psi)/2).$$

(8.4)

where c is a measure of the strength of the learning dynamic. Fig. 8.6 represents the potential wells of this function with ψ_{0} set to 0°, 180°, and 90°. The consequence of the operation of the learning dynamic is a stable relative phase minima at the environmentally specified phase pattern—0°, 180°, or 90°. As Zanone and Kelso point out, the shape of the consequent potential is a product of the competition or cooperation between the intrinsic dynamic and the dynamic necessary to produce the to-be-learned pattern. In Fig. 8.6, the wells for ψ = 0° and ψ = 180° have a greater concavity than that for ψ = 90°. In addition, the minima of the former two are more singularly defined points, whereas that of the latter is defined more as a

![Fig. 8.6 Potential function solutions of Equation 3 that represent the interaction of the control structure dynamic and the learning dynamic when the to-be-learned pattern is at 0°, 90°, and 180° (from left to right). The consequent concavity of each well denotes the amount of competition or cooperation between the intrinsic dynamics of bimanual phasing (Equation 2) and the dynamics required by the new pattern. See text. Adapted from Zanone and Kelso 1992.](image-url)
region. Both properties indicate that the evolution of the $\psi = 90^\circ$ attractor involves a competition between the intrinsic dynamic of the phasing control structure and the to-be-learned dynamics, whereas the evolution of attractors nearer to $\psi = 0^\circ$ and $180^\circ$ attractor involves more cooperation.

The modeling of the learning dynamic by Schöner (1989) suggests that when competition exists between the intrinsic and the to-be-learned dynamics, there is the possibility that the genesis of a new behavior attractor will annihilate attractor regimes in the original dynamics of the control structure and produce abrupt transitions to new attractive regions. The consequence of the new, emergent equations of constraint is that the original capabilities of the control structure may be eliminated. Zanone and Kelso provide evidence from their experiment that (a) subjects often demonstrated an abrupt acquisition of the $90^\circ$ relative phase pattern, and (b) after they had acquired this relative phase pattern, the stability of $180^\circ$ phasing was decreased. Although there were individual differences in these effects, the latter substantiates the idea that a restructuring of the control regime occurs, rather than just another control regime being added. The abrupt transition suggests that this was a process of dynamical change (e.g., a phase transition).

Generally, one can understand the problem of acquiring a skill as “how an extrinsically imposed task [goal] becomes internalized... so that [this goal state] acts as an intrinsic rather than an extrinsic constraint” (Shaw et al. 1992, 4). For example, how does the extrinsic constraint of an instructor’s movement become internalized by the learner? Zanone and Kelso suggest that the process of learning a new movement pattern is tantamount to transforming environmental information into memorized information. In a different language, the environmental constraint specified in the perceptual media can be used to transform the equation of constraint (in this case, Equation 2) employed by a dynamical control structure. Moreover, Schöner and colleagues provide a dynamical process (Equation 3) that describes this internalization of constraint.

What does this process correspond to in the activity of the organism? How does the equation of constraint get rewritten? One can think about the process as an exploration of a perceptual-motor work space, because the information being memorized at each point is specified in the proprioceptive information that the kinetics of the movement pattern being performed generate. Because the exploration of the work space is a goal-directed, intentional process, the operation of a learning dynamic such as that in Equation 3 can be interpreted as an intentional process that emerges from (a) having the goal to learn the new movement pattern, and (b) using (proprioceptive) information available that specifies how to transform the equation of constraint (i.e., information that specifies the parameterization of greatest stability) to produce the goal state. The learning process then has two descriptions—one intentional (just stated) and one deterministic (Equation 3). Are these not contradictory? The two descriptions are not contradictory if one countenances the possibility that intentional systems participate in deterministic dynamics, and their nondeterministic intentional behavior emerges from the sheer volume of the deterministic processes interacting across many levels of space/time (Shaw and Kinsella-Shaw 1988).

Learning a New Bimanual Frequency-Locking Pattern

Schmidt et al. (1992) is another example of a study that provides a dynamical analysis of the acquisition of an interlimb coordination pattern. In the Zanone and Kelso study, the movement pattern to be acquired was coordinating two limbs rhythmically moving at the same frequency (1:1 frequency lock) but with different phase locks or lags. In the Schmidt et al. study, the interlimb pattern to be acquired was the bimanual coordination of effectors moving at different frequencies. What was studied was the 2:1 frequency locking of weighted handheld pendulums swung from the wrist in the sagittal plane. The coordination of such wrist-pendulum systems in a 1:1 frequency lock has been studied to reveal the self-organizing principles underlying the coordination of rhythmic movements (e.g., Kugler and Turvey 1987; Turvey et al. 1988; Schmidt, Shaw, and Turvey 1993). The novelty of such a paradigm lies in the fact that the rhythmic units have well-defined frequency preferences as gravitational pendulums. This allows the experimenter to manipulate environmental constraints on the coordination—for example, how much the intrinsic frequencies of the rhythmic units are competing and how much the frequency competition between the units aids or opposes the coordination required by the task. In the Schmidt et al. experiment, the challenge was to acquire a 2:1 frequency lock with rhythmic units whose ratio of uncoupled frequencies was 1:1.

The intrinsic gravitational predilection of the rhythmic units made performing a 2:1 frequency lock more difficult than other 2:1 coordinations frequently encountered (e.g., finger tapping). The subject’s task was to find linkages between the laws of interlimb coordination control structures and the laws that govern the pendulums to be controlled. To put it another way, they were to find the equations of constraint that guide the interaction of these two systems. The task Schmidt et al. took was (a) to use a measure of mean frequency-locking ratio and stability of frequency locking to estimate the change in the dynamics across the learning sessions, and (b) to describe the property of the equation of constraint that was being tuned to produce this change in stability.

Subjects were instructed to oscillate the pendulum in the right hand so it was at twice the frequency of that in the left hand. They performed this coordination pattern for 20 trials in 12 sessions that were on average 1.5 weeks apart. Both the deviation from a perfect 2:1 frequency ratio (right/left) and standard deviation of this frequency ratio decreased asymptotically across the 12 sessions (fig. 8.7). These measures were used as indices of the topology underlying the dynamics’ potential function. The deviation from intended frequency lock indexes the average position of the potential well’s minima. Assuming a constant amount of stochastic noise emerges from the microstructure of the effector system, the standard deviation of the behavior can be used as an index of the concavity of the potential well governing the control structure dynamic producing the behavior: the less steep the sides of the well, the greater the standard deviation (Schöner 1989). The argument is that a stochastic perturbation of a given magnitude of force will travel through the force field of the potential well less far when the rate of change of the potential is less steep. Assuming these relations, the evolution of the attractor governing the 2:1 frequency lock can be estimated (fig. 8.8).

Presumably, the subjects are attuned to information in a perceptual-motor work space that specifies how the manipulation of certain parameters of the control structure’s equation of constraint will affect the control structure’s stability. But what are the parameters being manipulated to cause the optimization? Schmidt et al. examined the relative phasing of the two limbs to answer this question. Intuitively,
the 2:1 frequency-locking task requires specific relative movements of the two wrist-pendulum systems. Accordingly, Schmidt et al. examined a measure that captures the macroscopic space/time order of the 2:1 task—the pattern of oscillator phase velocities. The oscillator phase velocity is the change rate of its phase angle around its limit cycle. In perfectly 2:1 frequency-locked oscillators the phase velocity of the faster oscillator is always twice that of the slower oscillator; hence, the phase velocity ratio (PVR) will be a constant value of 2. By calculating the time series of the PVR for the learning trials, an index of the relative movements of the limbs was attained. Neither at the beginning nor at the end of their training were the relative movements like those of perfectly frequency-locked oscillators. The time series exhibited an oscillation about the criterial value of 2 with greater deviation in oscillation in the first compared with the last session. A spectral analysis of the PVR across the 12 sessions revealed a gradual evolution of spectral peaks at integer multiples of the frequency of oscillation (fig. 8.9). Such a consequent spectral pattern indicates that, although phase locking perhaps was never achieved, phase entrainment was. This is consistent with previous research on 1:1 frequency locking, which demonstrates that even when the rhythmic units are at the same tempo their relative phase is not at a constant value (i.e., for 1:1 frequency lock, PVR = 1 or \( \phi = \))

**Fig. 8.7** Representation of changes in stability in learning a 2:1 frequency-locking pattern with wrist-pendulum systems. Experimental mean data from four subjects representing the deviation from 2:1 frequency lock (top) and fluctuation in frequency-locking behavior (bottom) across 12 learning sessions.

**Fig. 8.8** Schematic representation of the evolution of the movement control structure's potential field across learning sessions. The position of the potential minima was estimated using the deviation from a 2:1 frequency lock, and the concavity of the attractive regions was estimated using the fluctuation in frequency-locking behavior. See text. Reprinted from Schmidt et al. 1992.

Just what is underlying these spectral peaks and why their topology changes under certain conditions is still a question. Two hypotheses have been ventured: (a) the peaks correspond to specific rhythmic subprocesses of perceptual constraint, for example, perceptual anchor points (Beek 1989; Schmidt et al. 1991), or (b) the peaks are a manifestation of the different oscillatory dynamics of the component oscillators—for example, the left hand is a harmonic oscillator and the right hand a van der Pol (Schmidt, Shaw, and Turvey 1993). Whichever explanation is forthcoming given the evolution of the PVR spectrum across learning trials, the case can be made that the subject learning to perform a 2:1 frequency-locking pattern is manipulating certain parameters of a dynamical equation of constraint about a specific phase entrainment pattern. The parameters may be becoming attuned to certain kinds of perceptual information needed for the entrainment pattern (Hypothesis 1) or may be the adjusting of the weights on the various friction and stiffness functions (Beek and Beek 1988) underlying the oscillatory behavior of the rhythmic units (Hypothesis 2).

The Schmidt et al. (1992) study demonstrates that acquiring a movement pattern can be understood in terms of learning the intrinsic lawful regularities of a movement control structure and how they must be reparameterized to produce the successful control of certain environmental objects (in this case, the pendulums) that have their own dynamics. This study highlights that, necessarily, the coordination dynamics underlying an action result from the interplay of organismic, environmental, and
Learning to Cascade Juggle

Another skill in which the dynamics of coordination are obviously written across the actor and the environment is juggling. In this activity, the actor must coordinate not only the constraint in the game of his/her system (strategy), but also the environment (balls). The ball residence (environment) movements of juggling, the movements of juggling, are under both temporal and spatial task constraints (Beek and van der Smagt 1991). Hence, the emergent control structure is written across both the actor and his/her environment, and the juggling itself is being described by the dynamical law of coupled oscillators defined by the interaction between the actor and the environment.

The frequency of oscillation is given by the equation:

\[ f = \frac{1}{2} \left( \frac{1}{T_1} + \frac{1}{T_2} \right) \]

where \( T_1 \) and \( T_2 \) are the times for each ball to return to the catching hand.

In an experiment in which 20 previously unskilled subjects were taught to juggle in 10 half-hour sessions, the frequency decreased to near 0 as the number of cycles juggled increased in the first 5 sessions. The second stage of juggling corresponds to optimizing the internal organization of the juggling hand cycle, that is, developing a 'faster' ratio of \( T_1/T_2 \). In the same experiment, Beek found that for the slow learners, the ratio of \( T_1/T_2 \) was greater than 0.75. In the same

The ratio of \( T_1/T_2 \) can be measured by an index of juggling skill, which is derived from Equation 5 using the experimentally acquired \( T_1 \) and \( T_2 \). In an experiment in which 20 previously unskilled subjects were taught to juggle in 10 half-hour sessions, the frequency decreased to near 0 as the number of cycles juggled increased in the first 5 sessions. The second stage of juggling corresponds to optimizing the internal organization of the juggling hand cycle, that is, developing a 'faster' ratio of \( T_1/T_2 \). In the same experiment, Beek found that for the slow learners, the ratio of \( T_1/T_2 \) was greater than 0.75. In the same

The ratio of \( T_1/T_2 \) can be measured by an index of juggling skill, which is derived from Equation 5 using the experimentally acquired \( T_1 \) and \( T_2 \). In an experiment in which 20 previously unskilled subjects were taught to juggle in 10 half-hour sessions, the frequency decreased to near 0 as the number of cycles juggled increased in the first 5 sessions. The second stage of juggling corresponds to optimizing the internal organization of the juggling hand cycle, that is, developing a 'faster' ratio of \( T_1/T_2 \). In the same experiment, Beek found that for the slow learners, the ratio of \( T_1/T_2 \) was greater than 0.75. In the same
75 at session 4 but not significantly different from .75 at session 7 or 10. Alternatively, the faster learners had already achieved t/h = .75 by session 4 but by session 10 their ratio was significantly less than .75 (t/h = .73). These results indicated that the slower learners reached stage 2 of learning to juggle between session 4 and session 7, and the faster learners reached it by session 4. Finally, the third stage of juggling was revealed only in the faster learner group in that they had a t/h ratio that was significantly less than the theoretically derived preferred ratio of .75 by session 10. This suggests that these jugglers could maintain a stable juggle with a degree of flexibility. The subjects had learned to modulate the mode locking and to explore the space around the mode lock at t/h = .75. In so doing, they could balance the stability of their performance with adaptability, the hallmark characteristic of the third stage of juggling.

Learning to juggle is tantamount to reconfiguring the intrinsic dynamics of the bimanual rhythmic movement control structure to function in accord with the required task constraints. Notice that in achieving this goal the actor has to satisfy the intrinsic constraints of the bimanual control structure, the environmental constraints of the balls, and the task constraints of the action. The dynamical process of constraint satisfaction involved here is identical in the abstract to that described in learning in connectionist machines (Churchland and Sejnowski 1989; Smolensky 1986). Satisfaction of the multiple constraints means the discovery of an equilibrium or steady state relation between the necessary constituents of an action. This process of constraint satisfaction underlies the evolution of the attractor layout of the control structure dynamics. However, the constraint satisfaction processes discussed here are at the macroscopic level of the eco-niche rather than that of the neural net. This is not to suggest that constraint satisfaction on the neural level doesn’t occur or is irrelevant, but only that it is part of a constraint satisfaction process that includes the properties of the action system and the environment.

SUMMARY

The dynamical theory of motor learning is based on the dynamical theory of motor control. Rather than provide a prescription for the organization of an action, this approach to motor control seeks to understand how action emerges based on generic dynamical principles that precipitate self-organization. The dynamical theory of motor control is similar in this respect to many connectionist theories that seek to give a principled account of the presence of behavioral order. Contrary to connectionist theories, the dynamical theory does not suppose that the nervous system is the propriety level of explanation of action. In addition to neural components, the organization of an action is constrained by components and dynamical processes at the level of the action system (e.g., coordinative structures) and at the level of the environment (e.g., the laws of perceiving and acting).

Instead of rules of action being encoded in the CNS, the dynamical theory of motor learning proposes that acquiring a skilled action requires becoming attuned to the physical laws that govern a dynamical control structure. These laws are not housed in a specific mechanism (like the CNS), but are distributed across all the components of the action system and environment, and as such, are (in a manner of speaking) incorporeal. Because perceptual information is available about the state of the dynamical control structure and how it changes over reparameterizations of the control structure, information exists in a perceptual-motor work space about the direction the reparameterizations should take to reach an optimally configured equation of constraint. Learning the laws that govern an effector system results from exploring this perceptual-motor work space with the intention of achieving a more stably configured control structure. This long time scale, intentional process can also be given a deterministic description in terms of the operation of a learning dynamic—a higher-order process of dynamical relaxation to an optimal state in which multiple constraints are uniquely satisfied so the functioning of the control structure can be referred to as skilled.

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